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Holographic Schwinger effect and chiral condensate in SYM theory

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ABSTRACT: We study the instability, for the supersymmetric Yang-Mills (SYM) theories, caused by the external electric field through the imaginary part of the action of the D7 probe brane, which is embedded in the background of type IIB theory. This instability is related to the Schwinger effect, namely to the quark pair production due to the external electric field, for the $SU(N_c)$ SYM theories. In this holographic approach, it is possible to calculate the Schwinger effect for various phases of the theories. Here we give the calculation for $\mathcal{N} = 2$ SYM theory and the analysis is extended to the finite temperature deconfinement and the zero temperature confinement phases of the Yang-Mills (YM) theory. By comparing the obtained production rates with the one of the supersymmetric case, the dynamical quark mass is estimated and we find how it varies with the chiral condensate. Based on this analysis, we give a speculation on the extension of the Nambu-Jona-Lasinio model to the finite temperature YM theory, and four fermi coupling is evaluated in the confinement theory.

KEYWORDS: AdS-CFT Correspondence, Brane Dynamics in Gauge Theories, D-branes, Holography and quark-gluon plasmas

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1 Introduction

The phenomenon related to flavor quarks, which is identified with the $\mathcal{N} = 2$ hypermultiplet, in supersymmetric Yang-Mills (SYM) theory has been studied by embedding D7 brane(s) as a probe in the D3 stacked background of the type IIB theory [1]–[12]. The profile of the D7 brane provides us information about chiral condensate of a quark in the dual $\mathcal{N} = 2$ SYM theory.

In this direction, the “electro-magnetic” properties of the system have been studied by imposing the external electro-magnetic field of $U(1)_B$ on the system, where the charge of the current corresponds to the baryon number (see refs. [5, 6]). Many properties of the system have been cleared in this setting through the D7 branes. Recently, in the SYM theory, the Schwinger effect has been studied according to the idea that the D7 brane action S_{D7} can be related to the Euler-Heisenberg Lagrangian $\mathcal{L}(E)$ as follows [14],

$$S_{D7} = - \int d^4x L(E), \quad (1.1)$$

where the internal space of the D7 world volume is integrated out and E denotes the external electric field imposed on the system. In this context, the Schwinger effect in SYM theory has been given by the imaginary part of this Lagrangian, $\text{Im}L(E)$ [14]–[16].

On the other hand, as shown in [5], this imaginary part can be removed by introducing an appropriate electric current which brings the system to a non equilibrium steady state. This fact implies that $\text{Im}L(E)$, which is given for a state without the electric current, is

regarded as the transition probability from a false vacuum without any current to the non equilibrium steady state with an appropriate current. So it may be reasonable to regard $\text{Im}L(E)$ as the pair production rate of the positive and negative charges, namely the quark and anti-quark. In this sense, this process can be considered as the Schwinger effect [17].

The pair production rate from (1.1) is obtained as follows. First, embed D7 probe brane in a given bulk which corresponds to the vacuum state of the dual theory. Then evaluate $\text{Im}L$ from (1.1) after imposing an external electric field E . $\text{Im}L(E)$ is obtained for $E \geq E_c$ where E_c is determined by the theory.

Here we remember the Schwinger's one-loop formula which is given in the four dimensional (4D) quantum electrodynamics (QED). In the case of the D7 brane embedded in the $\text{AdS}_5 \times S^5$ bulk, a hypermultiplet (one fermion and two scalars) is considered in the $\text{SU}(N_c)$ $\mathcal{N} = 2$ supersymmetric dual gauge theory. Thus for the hypermultiplet with the mass m , the Schwinger's formula would be considered by supposing the tunneling process as in the QED. The formula for $\text{Im}L$ obtained according to the method mentioned above, is not however equivalent to the Schwinger's formula due to the reason that the production rates are obtained via vacuum decay, which is not a tunneling.

We show that this lower bound E_c mentioned above is needed to remove the attractive quark potential, which increases linearly in the short range distance between the pair produced quark and the anti-quark in the SYM theory [2]. Then enough repulsive force coming from the external field is necessary to overcome this attractive force and to separate the pair produced quarks. Thus, we could find a stable electric current under an enough strong E . The value of E_c reflect the dynamics of the dual theory.

It is interesting to see the dynamical properties of the various $\text{SU}(N_c)$ gauge theories through the Schwinger effect given by (1.1). Our purpose is to investigate this point. The calculations are extended to the YM theories in the finite temperature deconfinement phase and in a zero temperature confinement phase.

The dynamical properties of these two theories are complicated and dependent on the parameters of the theories. Here we study how the production rate Γ depends on the chiral condensate $\langle \bar{\Psi}\Psi \rangle$ and its relation with the effective quark mass m_q^{eff} . As for the m_q^{eff} , we estimate it by comparing the production rate obtained for the supersymmetric theory, which is dual to $\text{AdS}_5 \times S^5$, with the one for the above two non-supersymmetric theories. The mass shift in the non-supersymmetric theories is measured through the pair production rate. Then, a speculation on the NJL model of QCD is given, and an extension of the NJL to finite temperature theories is proposed.

In [18], the pair production rate of the W bosons has been studied as a holographic Schwinger effect in terms of a probe D3 brane and a string in the D3 background. In this case, the production rate is obtained in terms of the tunneling process. An interesting point, in this calculation, is the existence of the second critical electric field which is needed for the tunneling in a theory of confining phase [19]. In the present case of D7/D3 system, this kind of production rate would be obtained from the real part of D7 action given above. We will discuss on this point very briefly in the article.

In the next section, we give a brief review of our D3/D7 brane model for a finite temperature Yang-Mills (YM) theory. In the section 3, how to calculate the production

rate of the quark pair by using (1.1) as a Schwinger effect is shown. In the section 4, the production rate is calculated for the YM theory whose vacuum state is in the finite temperature deconfinement phase. At first, the production rate is given for the case of the massive quark, and then the effect of the temperature and chiral condensate are examined. The effective quark mass is estimated by relating the production rate for the massive quark to the one obtained for the supersymmetric theory which has zero chiral condensate. Then the results are discussed by supposing the relation to the NJL model of QCD. In the next section, the parallel analysis is performed for the theory whose vacuum state is in the confinement and broken chiral symmetry phase. We find a good relation between the NJL model and our resultant formula for the effective quark mass. Summary and discussions are given in the final section.

2 D3/D7 model and D7 embedding

We study the Schwinger effect for the $\mathcal{N} = 4$ SYM theory coupled to $\mathcal{N} = 2$ hypermultiplet, whose holographic dual is given by D3/D7 branes system in the type IIB string theory.

Here, as a prototype of the model, we show the case of the finite temperature deconfinement phase. Its background metric is given by the AdS_5 Schwarzschild $\times S^5$, which is written as

$$ds^2 = \frac{r^2}{R^2} (-f^2(r)dt^2 + (dx^i)^2) + \frac{1}{f^2(r)} \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2, \quad (2.1)$$

where $R^4 = 4\pi\alpha'^2 N_c$ and

$$f(r) = \sqrt{1 - \left(\frac{r_T}{r}\right)^4}, \quad (2.2)$$

and the temperature T is given by $T = \frac{r_T}{\pi R^2}$. The embedding of the D7 brane is performed according to [20] by rewriting the six dimensional part of the above metric (2.1) as

$$\frac{1}{f^2(r)} \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 = \frac{R^2}{U^2} (d\rho^2 + \rho^2 d\Omega_3^2 + (dX^8)^2 + (dX^9)^2) \quad (2.3)$$

where

$$U(r) = r \sqrt{\frac{1 + f(r)}{2}}, \quad (2.4)$$

and

$$U^2 = \rho^2 + (X^8)^2 + (X^9)^2. \quad (2.5)$$

Then the DBI action of a D7-brane is given as

$$S_{D7} = -\tau_7 \int d^8\xi \sqrt{-\det(g_{ab} + 2\pi\alpha' F_{ab})}, \quad (2.6)$$

with the following induced metric g_{ab} ,

$$ds_8^2 = g_{ab} \xi^a \xi^b = \frac{r^2}{R^2} (-f^2(r)dt^2 + (dx^i)^2) + \frac{R^2}{U^2} ((1 + w'(\rho)^2) d\rho^2 + \rho^2 d\Omega_3^2). \quad (2.7)$$

We set $X^8 = w(\rho)$ and $X^9 = 0$ from the rotational symmetry of $X^8 - X^9$ -plane.

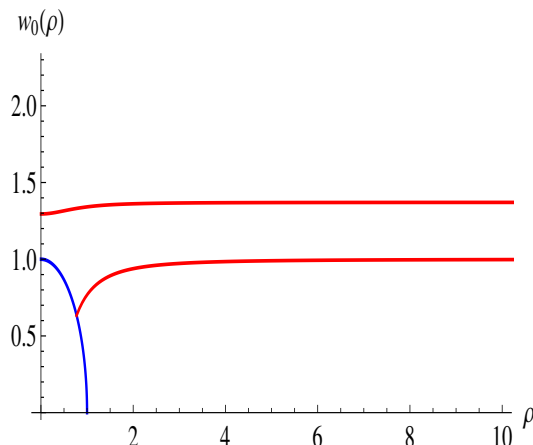


Figure 1. The typical embedding solutions $w_0(\rho)$ are shown for $r_T = \sqrt{2}$ and $R = 1$. The upper one is for $m_q = 1.37$ (Minkowski embedding), and the lower curve for $m_q = 1$ (Black Hole embedding).

We briefly review of the typical two types of embedding of the D7 brane, which are characterized by the profile function $w(\rho)$ given above. We denote it as $w_0(\rho)$ for the case of $F_{ab} = 0$. In this case, there is no gauge field ($F_{ab} = 0$) and the on-shell action of D7-brane is obtained by substituting $w_0(\rho)$ as

$$S_{D7} = - \int d^4x L_{D7}, \quad (2.8)$$

$$L_{D7} = 2\pi^2\tau_7 \int d\rho \mathcal{L}_{D7}, \quad \mathcal{L}_{D7} = \frac{r^4}{U^4} \rho^3 f(r) \sqrt{(1 + w_0'(\rho)^2)}. \quad (2.9)$$

We notice that L_{D7} , which is evaluated at on-shell with appropriate counterterms, is considered as the effective Lagrangian of the Yang-Mills theory with a quark.

The two typical solutions, which are called as the black hole (BH) embedding and the Minkowski embedding respectively, of $w_0(\rho)$ are shown in the figure 1. Here we give the following comments on these solutions.

- At large ρ , $w_0(\rho)$ behaves as

$$w_0(\rho) = m_q + \frac{c}{\rho^2} + \cdots, \quad (2.10)$$

we can get the value of the current quark mass m_q and the chiral condensate $c \equiv -\langle \bar{\Psi}\Psi \rangle$ respectively. So the effective Lagrangian is given as a function of m_q , T , and c .

- As shown in the figure 1, in the Minkowski embedding solutions, the D7-brane is off the horizon r_T . On the other hand, the black hole embedding solutions attach to the horizon.

3 Holographic Schwinger effect

External electric field. We add non-trivial gauge field in the D7 action to impose an external electric field E on the system considered in the previous section. We consider the following two cases with E .

(A) The first case is to find a stable state by imposing the external electric field E [5]. E is imposed on this system through A_a , which is defined as $F_{ab} = \partial_a A_b - \partial_b A_a$, as

$$\tilde{A}_x \equiv 2\pi\alpha' A_x = -Et + h(\rho), \quad (3.1)$$

and the other components are zero. In this case, \mathcal{L}_{D7} is written as

$$\mathcal{L}_{D7} = \frac{r^2 \rho^3}{U^4} \sqrt{f^2 r^2 U^2 h'(\rho)^2 + (f^2 r^4 - E^2 R^4)(1 + w'(\rho)^2)}, \quad (3.2)$$

where $h'(\rho) = \partial_\rho h(\rho)$. Then, the electric properties of the system under a constant electric field E have been studied by solving the equations of motion for $h(\rho)$ and $w(\rho)$ obtained from the above Lagrangian (3.2) [5–8, 21]. In this case, $h(\rho)$ is introduced to realize a non equilibrium steady state with an electric current j , which is defined as

$$\frac{\partial \mathcal{L}_{D7}}{\partial h'(\rho)} \equiv j. \quad (3.3)$$

(B) The second case is to set \tilde{A}_x as follows,

$$\tilde{A}_x = -Et, \quad (3.4)$$

instead of (3.1) [14]. In this case, the electric current is absent since we set as $h(\rho) = 0$. This setting implies that the electric field suddenly turned on at $t = 0$. As a result, we find a false vacuum which induces an imaginary part of L_{D7} given by (2.9). This imaginary part, $\text{Im} L_{D7}$, is considered as the transition rate to a stable state with a constant electric current j as given in (A). Then this is connected to the production rate of the electric charges, which becomes the source of this electric current j . This phenomenon is therefore related to the Schwinger effect. In fact, in this case, (3.2) is written as

$$\mathcal{L}_{D7} = \frac{R^2 r^2}{U^4} \rho^3 \sqrt{(1 + w'_0(\rho)^2) \left(\frac{r^4 f^2}{R^4} - E^2 \right)}, \quad (3.5)$$

where $w_0(\rho)$ denotes the solution of the equation of motion for $E = 0$, just before imposing E . This is proportional to $\sqrt{\frac{r^4 f^2}{R^4} - E^2}$, and then \mathcal{L}_{D7} becomes imaginary in an appropriate region of ρ where $\frac{r^4 f^2}{R^4} < E^2$ is satisfied. The lower bound of the electric field, $E = E_c$, is given as the minimum value of $r^4 f^2 / R^4$.

Then the production rate, Γ , of the quark-antiquark pair is obtained. It is defined as follows,

$$\Gamma \equiv \frac{\text{Im} L_{D7}}{2\pi^2 \tau_7} = \int_{\rho_{\min}}^{\rho_c} d\rho \frac{R^2 r^2}{U^4} \rho^3 \sqrt{(1 + w'_0(\rho)^2) \left(E^2 - \frac{r^4 f^2}{R^4} \right)}. \quad (3.6)$$

The upper limit ρ_c is determined by the equation, $\frac{r^4 f^2}{R^4} \Big|_{\rho=\rho_c} = E^2$. It is given by solving the following equations,

$$\rho_c = \sqrt{U_c^2 - w_0(\rho_c)^2}, \quad (3.7)$$

where

$$U_c = r_c \sqrt{\frac{1 + f(r_c)}{2}}, \quad (3.8)$$

and r_c is defined as

$$r_c^2 f(r_c)/R^2 = E. \quad (3.9)$$

As for ρ_{\min} , $\rho_{\min} = 0$ for the Minkowski embedding, and for the black hole embedding it is given as the point where the D7 brane touches at the horizon, namely $\rho_{\min}^2 + w_0^2(\rho_{\min}) = r_T^2/2$.

In the next, we explain how to calculate the above imaginary part.

- In (3.5) and (3.6), we notice that the profile function, $w_0(\rho)$, is the solution obtained before imposing the external E field. Namely, it is given in the previous section by (2.10).
- On the other hand, $w(\rho)$ in (3.2) of (A) represents the solution which should be obtained by solving the equation of motion derived from (3.2). So we notice $w(\rho) \neq w_0(\rho)$.
- Then it may be reasonable to regard the above imaginary part (3.6) as the transition probability from a destabilized state, $\{w_0(\rho), j = 0, E\}$, to an equilibrium steady state, $\{w(\rho), j \neq 0, E\}$, for a fixed $E(> E_c)$ [14].
- The constant current j must be supported by some free charges. They should be generated by the Schwinger effect from the vacuum state. In this sense, Γ could be regarded as the production rate of quark and anti-quark pair. Further, L_{D7} could be regarded as the Euler-Heisenberg Lagrangian for $\mathcal{N} = 4$ SYM theory coupled to $\mathcal{N} = 2$ hypermultiplet.

As mentioned above, m_q and c are already included in the above Lagrangian (3.5). To find the production rate, the remaining work is to perform the integration of (3.6) with respect to ρ over an appropriate range.

E dependence of Γ and physical quantities. The above expression of Γ contains various parameters, T , m_q , c and E . As for the E dependence, it is absorbed in the dimensional parameters in Γ , for example as $\tilde{r} = r/\sqrt{E}$. The dimensionful factor E^2 is separated out as a prefactor to fix the dimension of Γ . Namely, we can rewrite Γ as,

$$\frac{\Gamma}{2\pi^2\tau_7} \equiv E^2 \gamma(\tilde{m}_q, \tilde{T}), \quad (3.10)$$

$$\gamma(\tilde{m}_q, \tilde{T}) = \int_{\tilde{\rho}_{\min}}^{\tilde{\rho}_c} d\tilde{\rho} \frac{R^2 \tilde{r}^2}{\tilde{U}^4} \tilde{\rho}^3 \sqrt{(1 + \tilde{w}'_0(\tilde{\rho})^2) \left(1 - \frac{\tilde{r}^4 f^2}{R^4}\right)}, \quad (3.11)$$

where $\tilde{m}_q = m_q/\sqrt{E}$, $\tilde{T} = T/\sqrt{E}$, and $f = \sqrt{1 - (\tilde{r}_T/\tilde{r})^4}$.

In this case, γ is normalized as

$$\gamma(0,0) = \frac{R^4\pi}{8}. \quad (3.12)$$

In the above expression (3.10), the dimensionful factor E^2 is factored out, and the remaining E s are absorbed into $\gamma(\tilde{m}_q, \tilde{T})$ in terms of the newly defined variables, \tilde{m}_q, \tilde{T} , etc. Then, the dependences on \tilde{m}_q and \tilde{T} can be seen from m_q and T dependence of γ for a fixed E . So it is enough to examine Γ for one value of E to see the m_q and T dependence. We perform the analysis for a fixed value of E by varying other parameters, m_q and T , hereafter. As for the E dependence, we discuss in the last section.

4 Deconfining chiral symmetric phase

As shown above, Γ depends on $w_0(\rho)$, which reflects the chiral symmetry of the vacuum state before imposing the external E . At first, we examine the case of the chiral symmetric phase given by the metric (2.1). While, for $m_q > 0$, w_0 is given in the form of (2.10) with negative c , $c = 0$ is found for $m_q = 0$. Then we can say that the chiral symmetry is realized in this phase.

4.1 Quark mass dependence at $T = 0$, $\text{AdS}_5 \times S^5$ limit

Before considering the metric (2.1) for arbitrary T , we study m_q dependence at $T = 0$, namely the $\text{AdS}_5 \times S^5$ limit. In this case, w_0 is given as a constant (and $c = 0$),

$$w_0 = m_q, \quad (4.1)$$

for any m_q as shown in [13]. This state is supersymmetric, and Γ is expressed as

$$\Gamma(m_q, T = 0) \equiv \Gamma_0(m_q), \quad (4.2)$$

where E is also included in Γ , but it is abbreviated for simplicity. Then, (3.6) becomes

$$\Gamma_0(m_q) = \int_0^{\rho^*} d\rho \, \rho^3 \sqrt{\frac{R^4 E^2}{r^4} - 1}, \quad \rho^* = \sqrt{R^2 E^2 - m_q^2}. \quad (4.3)$$

This is evaluated as

$$\Gamma_0(m_q) = \frac{R^4 E^2}{2} I(\theta_0), \quad (4.4)$$

where

$$I(\theta) = \frac{\pi}{4} - \frac{\theta}{2} + \frac{1}{2} \sin \theta \left(\cos \theta - \log \left(\frac{1 + \cos \theta}{1 - \cos \theta} \right) \right), \quad (4.5)$$

and

$$\sin \theta_0 = \frac{m_q^2}{R^2 E} \left(= \frac{\tilde{m}_q^2}{R^2} \right). \quad (4.6)$$

We notice that there is a lower bound of E ,

$$E \geq \frac{m_q^2}{R^2} (= E_c), \quad (4.7)$$

to have a finite Γ since $\sin \theta_0 \leq 1$ or ρ^* should be real and finite.

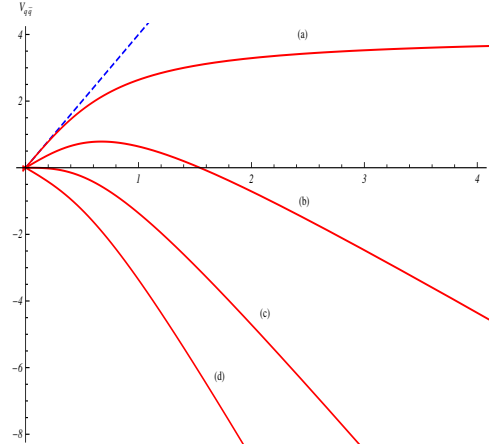


Figure 2. Quark-antiquark potentials $V_{q\bar{q}}$ defined in (4.8) are shown as functions of the distance l between quark and antiquark for $m_q = 2$, $R = 1$ and various E . The curves (a), (b), (c) and (d) are for $E = 0, 2, 4$, and 6 respectively. Here $r_{\max} = w_0(0) = m_q = 2$ and $E_c = m_q^2/R^2 = 4$. The dashed blue line represents the tangential line $m_q^2 l/R^2$ at the origin for $V_{q\bar{q}}$ with $E = 0$.

To understand the above production rate, we consider the pair produced quark-antiquark total potential $V_{q\bar{q}}$ [18] for various values of E . This is defined as

$$V_{q\bar{q}} = U_{q\bar{q}} - El \quad (4.8)$$

where $U_{q\bar{q}}$,¹ is the quark-antiquark potential at a distance l without the electric field E . The potential $U_{q\bar{q}}$ is obtained from the action of a string whose endpoints are on the D7-brane [2]. The endpoint, r_{\max} , can be taken at the various position on the D7-brane and the lowest value of r_{\max} corresponds to m_q . In the figure 2, we show the case of $r_{\max} = m_q$. The total potential $V_{q\bar{q}}$ are shown for $E = 0$ ((a)), $0 < E < E_c$ ((b)), $E = E_c$ ((c)), and $E > E_c$ ((d)) respectively.

We could find from the potential for $E = 0$ (curve (a) in the figure 2) that the bound value, m_q^2/R^2 , of (4.7) corresponds to the tension of the quark and anti-quark potential $V_{q\bar{q}}$ observed at very short distance as shown in the figure 2 by the dotted line. This tension is responsible for constructing a quark and anti-quark bound state, a meson [2]. At large distance, it behaves like a Coulomb potential and no linear potential is observed. So (4.7) is a sufficient condition to remove the attractive force, which is responsible to make a bound state of the pair produced quark and anti-quark, from $U_{q\bar{q}}$. In fact, for $E \geq E_c$, we find

¹ $U_{q\bar{q}}$ and l can be calculated from the on-shell Nambu-Goto action of the string whose endpoints are on the D7-brane [2], and they are given for $\text{AdS}_5 \times S^5$ background as

$$U_{q\bar{q}} = \frac{1}{\pi\alpha'} \int_{r_{\min}}^{r_{\max}} dr \frac{1}{\sqrt{1 - (r_{\min}/r)^4}},$$

$$l = 2R^2 \int_{r_{\min}}^{r_{\max}} dr \frac{1}{r^2 \sqrt{(r/r_{\min})^4 - 1}},$$

where r_{\min} is the bottom point of the string, and r_{\max} is the position of the string endpoints on the D7-brane.

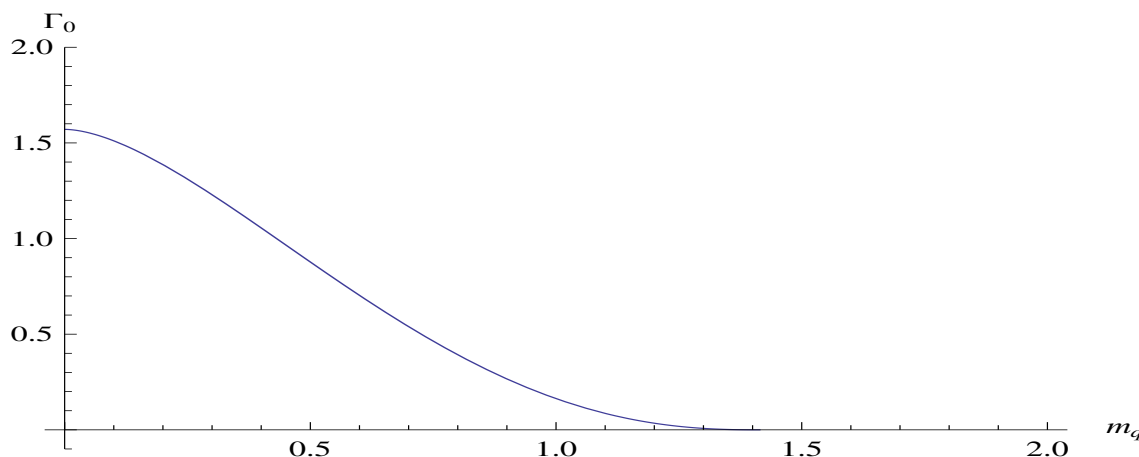


Figure 3. $\Gamma_0(m_q)$ is shown for $E = 2$ and $R = 1$. The curve represents the formula (4.3).

$V_{q\bar{q}} \leq 0$ for whole range of l as shown by the curves (c) and (d). Thus, in the case of (4.7), no tunneling process is needed to make a steady current.

As a result, we could say that $\Gamma_0(m_q)$ given by (4.3) represents the probability of the transition from a false vacuum with $j = 0$ to an equilibrium steady state with a finite j for a given $E(\geq \frac{m_q^2}{R^2})$. The latter state is obtained according to the way given in (A) in the previous section. Then, our result, the formula (4.4), is shown in the figure 3 for $E = 2$.

In the figure 3, non-zero Γ is obtained for $m_q \leq \sqrt{ER}(= \sqrt{2})$. For the case of $m_q > \sqrt{ER}$, the vacuum decay does not occur since the imposed E is less than the critical value E_c . Namely, for $0 < E < E_c$, the attractive potential remains as shown by the curve (b) in the figure 2. Then in this case, a tunneling process would be needed to produce free quark pairs. The tunneling process would be important in this case. We will give its analysis in a future work. Here, the analysis is restricted to the case of (c) and (d) of the figure 2.

4.2 Temperature dependence

Now we study the temperature dependence of the production rate, which is denoted as

$$\Gamma(T, m_q) \equiv \Gamma_T(m_q).$$

At finite T , we could see the similar behavior of $V_{q\bar{q}}$ to the one of the $T = 0$ case given above. In the case of Minkowski embedding, the quark-antiquark potential $V_{q\bar{q}}$ as shown in figure 4 disappears at the finite distance l by the thermal screening. Except for this point, the behavior of $V_{q\bar{q}}$ at small l is similar to the case of $T = 0$. As for the case of BH embedding, $V_{q\bar{q}}$ behaves as the curves (c) and (d) in figure 4 since $E_c = 0$ as shown below.

Case of $m_q = 0$. At first, we consider the case of $m_q = 0$. In the present case, the system is in a phase where the chiral symmetry is restored. In fact, $w = 0$ then $c = 0$ at any T . This implies that Γ would be independent of T . We show this point below.

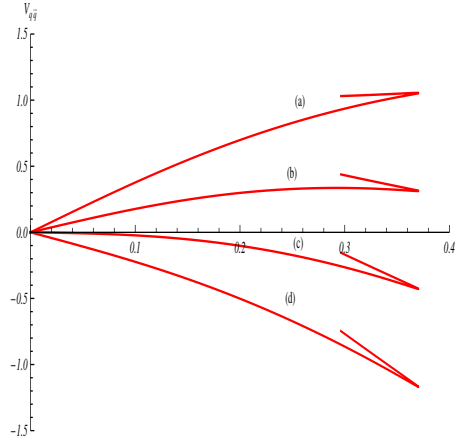


Figure 4. Quark-antiquark potential $V_{q\bar{q}}$ at finite temperature with $T = 1/\pi$ is calculated as a function of the distance l between quarks for $E = 0, 2, 4, 6$ as (a), (b), (c) and (d) respectively, where $R = 1$, $r_{\max} = w_0(0) = 2$ and $E_c = 4$.

For $m_q = 0$, $w = w' = 0$, $c = 0$ and

$$U = \rho, \quad r = \rho \sqrt{1 + \frac{r_T^4}{4\rho^4}}. \quad (4.9)$$

Then we obtain

$$\Gamma_T(0) = R^2 \int_{\rho_{\min}}^{\rho_{\max}} \frac{d\rho}{\rho} r^2 \sqrt{E^2 - \frac{r^4 f^2}{R^4}}, \quad (4.10)$$

where $\rho_{\min} = r_T/\sqrt{2}$ and $\rho_{\max} = \left\{ ER^2 \left(1 + \sqrt{1 + 4\rho_{\min}^4/(ER^2)^2} \right) / 2 \right\}^{1/2}$. We can perform the above integration, and we arrive at

$$\Gamma_T(0) = \frac{\pi R^4}{8} E^2. \quad (4.11)$$

It is noticed that the above result is independent of T . In fact (4.11) is the same value as (4.4) with $m_q = 0$,

$$\Gamma_0(0) = \frac{R^4 E^2}{2} I(0) = \frac{\pi R^4}{8} E^2. \quad (4.12)$$

Case of $m_q > 0$. In this case, the lower bound E_c exists. An example is shown in the figure 5 for $m_q = 1$ with $R = 1$. As seen from the figure 5, E_c changes drastically near $T_c \sim 0.35$, where the type of embedding changes from Minkowski to BH type. For $T > T_c$, BH embedding is realized and there is no stable meson in this phase since the attractive force at short distance disappears due to the thermal screening. Then we find $E_c = 0$ for $T > T_c$. The transition temperature, T_c , depends on the current quark mass m_q .

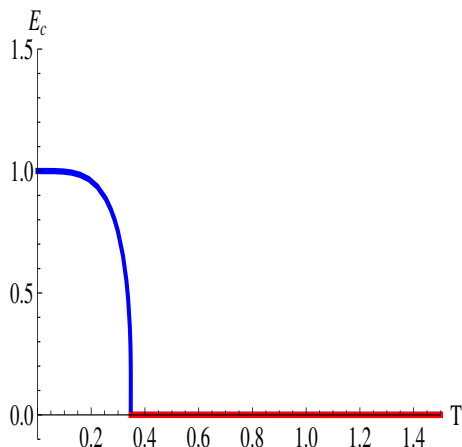


Figure 5. The relation between E_c and T for $m_q = 1$ with $R = 1$. The blue curve is obtained by Minkowski embedding solutions and red line is given by BH embedding solutions.

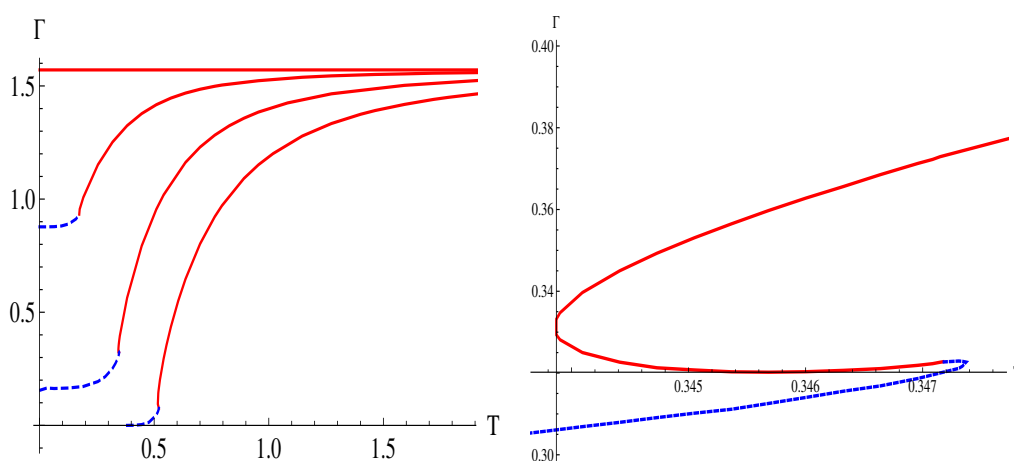


Figure 6. The left figure shows the relation between Γ and T with $E = 2$ and $m_q = 0, 0.5, 1, 1.5$ from the above. The dashed blue curves are given by the Minkowski embedding solutions and the solid red curves are obtained by the BH embedding solutions. For $m_q = 1.5$, Γ has a finite value for $T \geq 0.38$. The right figure shows the extended region near the transition point with $m_q = 1$. We can see the first order phase transition between Minkowski embedding and BH embedding.

Numerical results of Γ . In the figure 6, four results of numerical estimation of $\Gamma_T(m_q)$ are shown for $m_q = 1.5, 1.0, 0.5, 0$ with $E = 2$.

From this figure 6, we find a first order transition of Γ at $T = 0.5, 0.34$ and 0.22 for the cases of $m_q = 1.5, 1.0$ and 0.5 respectively. The transition temperature depends on the value of m_q . At this point, the embedding form of the D7 brane transits from the “Minkowski embedding” to the “BH embedding”. This is a well known phase transition observed in the holographic SYM theory. Near this point, Γ increases rapidly and approaches to the high temperature limit more slowly at large T .

Figure 6 also shows that $\Gamma_T(0)$ is independent of T as shown in (4.11), and $\Gamma_T(m_q)$ at $m_q = 1.5, 1.0, 0.5$ approach to $\Gamma_T(0)$ at $T \rightarrow \infty$. We will confirm this point with the analytic calculation as follows.

We evaluate the upper limit, $\Gamma_T(m_q)|_{T \rightarrow \infty} \equiv \Gamma_\infty(m_q)$. At large T , $\Gamma_T(m_q)$ is approximated as

$$\Gamma_\infty(m_q) = \int_1^{x^*} dx F(x), \quad (4.13)$$

$$F(x) = \left(\frac{U_T R}{2}\right)^2 \frac{1+x}{x\sqrt{x}} \sqrt{E^2 - \left(\frac{U_T}{R}\right)^4} f(x), \quad (4.14)$$

$$f(x) = x \left(1 - \frac{1}{x}\right)^2, \quad x = \left(\frac{U}{U_T}\right)^4, \quad (4.15)$$

where $w'(\rho)$ is approximated as being negligible small and

$$U_T = \frac{r_T}{\sqrt{2}}, \quad x^* = \alpha \left(1 + \sqrt{1 - \frac{1}{\alpha^2}}\right), \quad (4.16)$$

$$\alpha = 1 + \frac{R^4 E^2}{2U_T^4}. \quad (4.17)$$

Then we find

$$\Gamma_T(m_q)|_{T \rightarrow \infty} = \frac{R^4 E^2 \pi}{8} \left(1 - \frac{4R^2}{3\pi U_T^2} E + O(1/U_T^4)\right) \Big|_{T \rightarrow \infty} = \frac{\pi R^4}{8} E^2. \quad (4.18)$$

This result is independent of m_q and equivalent to $\Gamma_0(0)$ as shown in (4.12). The limiting value of Γ for all m_q approaches to this value at $T \rightarrow \infty$ as shown in figure 6.

Effective mass m_q^{eff} . As mentioned above, in the present case, we could find the chiral condensate c , which is negative, $c < 0$. Its value depends on m_q and T . Then, c decreases from $c = 0$ at $T = 0$ with increasing T monotonically. When we remember a simple NJL formula, (4.23) with $f(c) = 1$, we may expect that the effective quark mass m_q^{eff} would be suppressed from the current quark mass m_q , namely $m_q^{\text{eff}} < m_q$. Here we notice that the sign of c is opposite to the chiral symmetry broken phase. It may be obtained after a calculation of the self-energy with full quantum correction. It is however difficult to derive m_q^{eff} at finite temperature from the 4D SYM field theory side.

We propose a way to estimate m_q^{eff} by using an ansatz that the dynamical effects of the temperature are absorbed into m_q^{eff} and the production rate is replaced by the one given for the $\text{AdS}_5 \times S^5$ as given below.

This ansatz is based on the following speculation. For the case of $\text{AdS}_5 \times S^5$, the mass of the pair created quark receives no correction from the gauge interaction due to the preserved supersymmetry. For non-supersymmetric cases studied here for the finite temperature phase (and for the confinement phase in the below), on the other hand, the quark mass is modified to m_q^{eff} by the correction. The quark potential is similar to the case of $\text{AdS}_5 \times S^5$ due to the imposed $E(> E_c)$. So the difference is reduced to the quark mass. According to this consideration, the value of m_q^{eff} is obtained by comparing Γ for the non-conformal case with the one for the conformal case at $T = 0$.

Thus, under this ansatz, the production rate Γ at $T(\neq 0)$ can be related to Γ at $T = 0$, which is obtained for $\text{AdS}_5 \times S^5$ bulk, as follows,

$$\Gamma(m_q, T) \equiv \Gamma_T(m_q) = \Gamma_0(m_q^{\text{eff}}), \quad (4.19)$$

where m_q^{eff} is a function of m_q and T (or c). As shown in the figure 6, the T -dependence of $\Gamma_T(m_q)$ for fixed m_q is read as

$$\Gamma_0(m_q) \leq \Gamma_T(m_q) \leq \Gamma_\infty(m_q), \quad (4.20)$$

and from (4.12) and (4.18) we also find

$$\Gamma_\infty(m_q) = \Gamma_0(0). \quad (4.21)$$

Namely, the value of $\Gamma_T(m_q)$ is bounded between $\Gamma_0(m_q)$ and $\Gamma_0(0)$. Therefore, in the whole range of the temperature, $0 \leq T \leq \infty$, the value of m_q^{eff} could be found between m_q and zero by using the relation (4.19). While the equality (4.21) can be shown analytically by (4.12), the inequality (4.20) on the other hand assured by the numerical calculation as shown in the figure 6.

Here we give a comment on the relation of our method to determine m_q^{eff} and a way to use the on-shell action of a string which connects the D7 brane and the horizon at finite temperature [22]. The result of the latter method is in general different from ours. This discrepancy is clear in the case of BH embedding, where the method of the string action leads to $m_q^{\text{eff}} = 0$. In our case, however, it is finite, and $m_q^{\text{eff}} = 0$ is found only at the limit of the infinite temperature.

The reason of this disagreement is in the fact that, in our calculation of $\Gamma_T(m_q)$, various positions of r on the D7 brane, where the pair production occurs, are taken into account of. In other words, Γ is obtained by integrating over ρ from ρ_{\min} to ρ_c . The effective quark mass depends on the position of ρ . Then, after an average of these various positions for the effective quark mass, we have arrived at our result. At $T = \infty$, $m_q^{\text{eff}} = 0$ is obtained since the position is restricted to the one on the horizon.

T and c dependence of m_q^{eff} . Then the c and T dependences of m_q^{eff} are obtained by using (4.19). The results are shown for $m_q = 1$ and $E = 2$ in the figure 7. As expected, the effective mass decreases with T rapidly near the transition point ($T_c \sim 0.35$) and it slowly decreases in the region of large T . Then $m_q^{\text{eff}} = 0$ is realized in the limit of $T = \infty$.

At the same time, m_q^{eff} is also plotted as a function of the chiral condensate c , and it is shown in the left figure of the figure 7. Near the transition point from the Minkowski to the BH type embedding, it changes very rapidly.

4.3 m_q^{eff} and NJL coupling

In general, the effective mass of the quark is intimately related to the chiral condensate $-\langle \bar{\Psi}\Psi \rangle \equiv c$ and then it could be expressed as a function of c . It is important to find a precise form of the effective mass as $m_q^{\text{eff}}(c)$ for a fixed m_q . This kind of analysis is usually performed in terms of the Nambu-Jona-Lasinio (NJL) model [23, 24] for QCD,

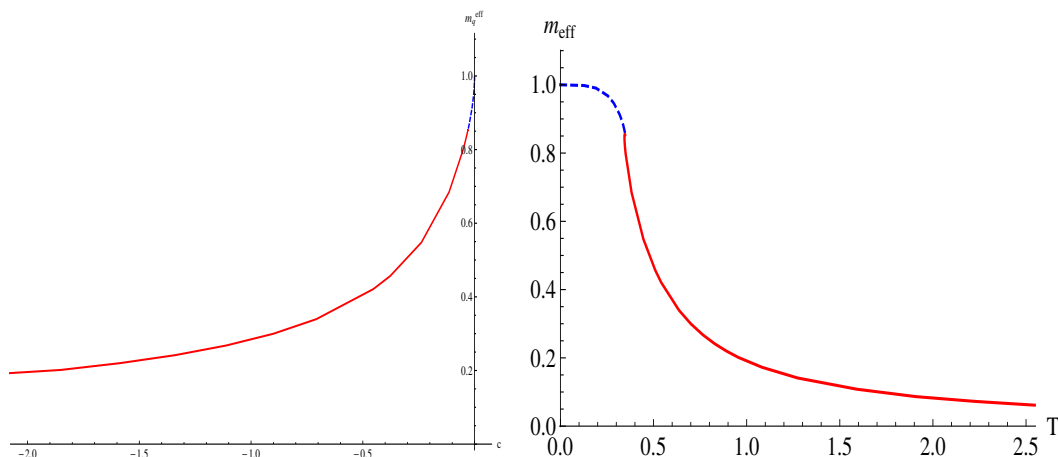


Figure 7. The c (Left) and T (Right) dependences of m_q^{eff} for $m_q = 1$, $E = 2$ and $R = 1$. The dashed blue curve is obtained by Minkowski embedding solutions and solid red line is given by BH embedding solutions.

where $m_q^{\text{eff}}(c)$ is obtained by supposing the effective Lagrangian of the quark with multi quark coupling terms.

In the present case, we are considering the $\mathcal{N} = 2$ SYM theory. At $T = 0$, we find $c = 0$ in this theory due to the supersymmetry. On the other hand, at finite temperature, we find negative c . The problem is to see how our results are understood from a NJL model for the $\mathcal{N} = 2$ SYM theory at finite temperature. This is simply an extension of NJL to the high temperature phase.

The NJL model is set as follows [25],

$$L_{\text{NJL}} = \bar{\Psi} (i\gamma^\mu \partial_\mu - m_q) \Psi + g_s (\bar{\Psi} \Psi)^2 + \dots \quad (4.22)$$

The details of this model are not discussed. We consider the simplest case of this model to compare with our holographic results.

While it would be possible to consider the various types of condensates of the fermi field, we do not consider them here since they would not contribute to the quark mass. Then, in the present analysis, the terms like $(\bar{\Psi} i\gamma_5 \Psi)^2$ are neglected. By considering the mean field approximation and taking into account of the higher order terms with respect to $\bar{\Psi} \Psi$, the effective quark mass would be obtained in the form,

$$m_q^{\text{eff}} = m_q + 2g_s c f(c), \quad (4.23)$$

where g_s is taken as a constant, and $f(c)$ denotes a function of c . In this case the higher order terms of c are absorbed in $f(c)$. For the case of $m_q = 1$ and $E = 2$, $f(c)$ is shown in the figure 8, and we have a rough estimation,

$$2g_s f(c) = \frac{6.25}{1 - 8.3c}. \quad (4.24)$$

This implies that we need infinite series of c ($1/c$) to explain this behavior at small (large) c . In any case, our result would give an important clue to understand the dynamics of the theory at finite temperature.

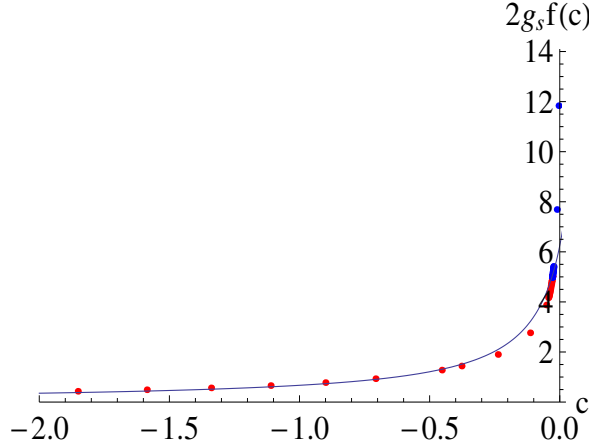


Figure 8. The relation between $2g_s f(c)$ in (4.23) and c for $m_q = 1$, $E = 2$ and $R = 1$ for the finite temperature case. The solid curve represents $2g_s f(c) = 6.25/(1 - 8.3c)$.

5 Confining phase

We consider a holographic theory which is in a chiral symmetry broken and quark confining phase. This is realized by adding a non-trivial dilaton, which corresponds to the vacuum with gauge condensate $\langle F_{\mu\nu} F^{\mu\nu} \rangle$ parametrized by r_0^4 . The bulk background, dual to confining gauge theory considered here, is expressed as [21],

$$\begin{aligned} ds_{10}^2 &= G_{MN} dX^M dX^N \\ &= e^{\Phi/2} \left\{ \frac{r^2}{R^2} A^2(r) (-dt^2 + (dx^i)^2) + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \right\}, \end{aligned} \quad (5.1)$$

in the string frame. $A(r)$ and the dilaton Φ are given by

$$A(r) = \left(\left(1 - \left(\frac{r_0}{r} \right)^8 \right) \right)^{1/4}, \quad e^{\Phi} = \left(\frac{(r/r_0)^4 + 1}{(r/r_0)^4 - 1} \right)^{\sqrt{3/2}}, \quad (5.2)$$

respectively. We should notice that this configuration has a singularity at the horizon $r = r_0$. So we can not extend our analysis to near this point. This difficulty would be resolved by introducing higher curvature contributions.

Fortunately, all the embedding solutions used here avoid the singularity for any region of the parameter which we used. This would be reasonable since a finite solution can not be defined at any singular point of the background.

The extra six dimensional part of the above metric (5.1) is rewritten as,

$$\frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 = \frac{R^2}{r^2} (d\rho^2 + \rho^2 d\Omega_3^2 + (dX^8)^2 + (dX^9)^2), \quad (5.3)$$

where $r^2 = \rho^2 + (X^8)^2 + (X^9)^2$. Then we obtain the induced metric for D7 brane,

$$ds_8^2 = e^{\Phi/2} \left\{ \frac{r^2}{R^2} A^2 (-dt^2 + (dx^i)^2) + \frac{R^2}{r^2} ((1 + (\partial_\rho w)^2) d\rho^2 + \rho^2 d\Omega_3^2) \right\} \quad (5.4)$$

$$= e^{\Phi/2} \tilde{G}_{ab} d\xi^a d\xi^b, \quad (5.5)$$

where we take as $X^8 = w(\rho)$ and $X^9 = 0$. The suffices a and b run from 0 to 7.

By taking the gauge field as $\tilde{A}_x(\rho, t) = -Et + h(\rho)$, we arrive at the following D7 brane action [21],

$$S_{D7} = -2\pi^2\tau_7 \int d^4x d\rho \rho^3 \frac{R}{r} e^{\Phi/2} \sqrt{Pe^\Phi - Q}, \quad (5.6)$$

$$P = |\tilde{G}_{00}|\tilde{G}_{xx}\tilde{G}_{\rho\rho}, \quad Q = \tilde{G}_{\rho\rho}\dot{\tilde{A}}_x^2 - |\tilde{G}_{00}|\tilde{A}'_x{}^2, \quad (5.7)$$

where $\tilde{G}_{MN} = e^{-\Phi/2}G_{MN}$. At first, we solve the equation of motion of $\tilde{A}'_x = h'$ as

$$e^{\Phi/2} \frac{\rho^3}{r} \frac{|\tilde{G}_{00}|\tilde{A}'_x}{\sqrt{Pe^\Phi - Q}} = J, \quad (5.8)$$

where J denotes a constant and it corresponds to the electric current,

$$J = \langle J_x \rangle. \quad (5.9)$$

As for the solutions with and without J , they are seen in [21]. The situation is similar to the above finite temperature case. By setting J as an appropriate value, we find a stable solution for $w(\rho)$ for a given E .

As in the case of the finite temperature theory, the imaginary part of the D7 action for a finite $E(\geq E_c)$, is given by setting $J = 0$ as

$$\Gamma = \frac{\text{Im } \mathcal{L}}{2\pi^2\tau_7} = \int_0^{\rho_c} d\rho \rho^3 A^2 e^{\Phi/2} \left(\frac{R}{r}\right)^2 \sqrt{1 + w'_0(\rho)^2} \sqrt{E^2 - A^4 e^\Phi \left(\frac{R}{r}\right)^4}, \quad (5.10)$$

where

$$\rho_c = \sqrt{r_c^2 - w_0^2(0)}, \quad (5.11)$$

and r_c is defined as

$$A^4 e^\Phi \left(\frac{R}{r}\right)^4 \Big|_{r=r_c} = E^2, \quad (5.12)$$

where $E \geq E_c$ and the r_0 dependence of E_c is shown in the figure 9.

Furthermore we notice the following points.

- The profile function $w_0(\rho)$ used in the eq. (5.10) is the one obtained from the D7 brane embedded before imposing the external electric field E . The equation of motion for $w_0(\rho)$ is therefore obtained from the following D7 Lagrangian,

$$\mathcal{L}_{D7}^{(0)} = -2\pi^2\tau_7 \int d\rho \rho^3 A^4 e^\Phi \left(\frac{R}{r}\right)^4 \sqrt{1 + w'_0(\rho)^2}. \quad (5.13)$$

- The second point is that there is a lower bound of E in order to have the above imaginary part. It is given as

$$E^2 \geq \frac{r^4 A^4 e^\Phi}{R^4} \Big|_{r=r_c} \geq \frac{r^4 A^4 e^\Phi}{R^4} \Big|_{r=w_0(0)} \equiv E_c^2. \quad (5.14)$$

Notice that there is no BH embedding in the present confining phase. Then the infrared end point of $w_0(\rho)$ is given by $w_0(0)$.

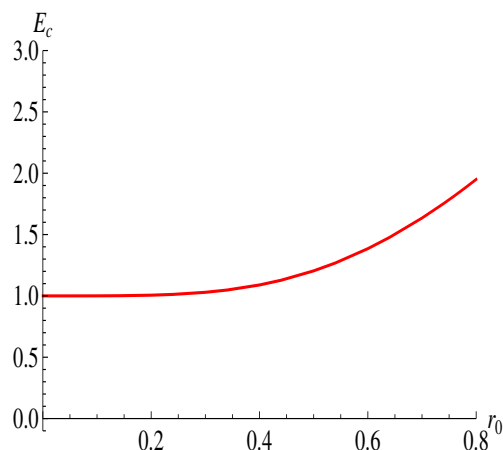


Figure 9. Relation between the critical electric field E_c and r_0 for $m_q = 1$ with $R = 1$.

In order to make free quark pair, it is necessary to overcome the confining force τ_{QCD} , which is the tension at large distance, by the external electric force E . In the present model, τ_{QCD} is given by

$$2\pi\alpha'\tau_{\text{QCD}} \equiv \tilde{\tau}_{\text{QCD}} = \frac{r^2 A^2 e^{\Phi/2}}{R^2} \Big|_{r=r^*}, \quad (5.15)$$

where r^* denotes the minimum point of $r^2 A^2 e^{\Phi/2}(r)$.

This is compared to the tension at short distance, τ_{SD} , which is given by

$$2\pi\alpha'\tau_{\text{SD}} \equiv \tilde{\tau}_{\text{SD}} = \frac{r^2 A^2 e^{\Phi/2}}{R^2} \Big|_{r=w_0(0)}, \quad (5.16)$$

and E_c is equivalent to $\tilde{\tau}_{\text{SD}}$ of the quark-antiquark potential calculated by the string whose endpoints are at $r_{\text{max}} = w(0)$ from (5.14). Noticing $r^* < w_0(0)$, we find

$$\tau_{\text{QCD}} < \tau_{\text{SD}}. \quad (5.17)$$

Then we find that (5.14) is sufficient to remove the binding force between the pair produced quark and anti-quark. Then the tunneling process is absent. This point is explained below in terms of the quark-antiquark potential $V_{q\bar{q}}$ given by (4.8).

In confining case, $V_{q\bar{q}}$ is described by several curves, (a) \sim (e), in figure 10. Here (a), (b), (c), (d), (e) denotes the $V_{q\bar{q}}$ with various E which satisfies $E = 0$, $E = \tilde{\tau}_{\text{QCD}}$, $\tilde{\tau}_{\text{QCD}} < E < \tilde{\tau}_{\text{SD}}$, $E = \tilde{\tau}_{\text{SD}}$ and $E > \tilde{\tau}_{\text{SD}}$ respectively. For $0 \leq E \leq \tilde{\tau}_{\text{SD}}$ ((a) and (b)), we find $V_{q\bar{q}} > 0$ for all region of l . Thus, we cannot find the free quark pair in this case. For $\tilde{\tau}_{\text{QCD}} < E < \tilde{\tau}_{\text{SD}}$ ((c)), free quarks can be produced by the tunneling process. For $\tilde{\tau}_{\text{SD}} = E_c \leq E$ ((d) and (e)), $V_{q\bar{q}}$ becomes negative in all region of l . Thus, for (5.14), free quarks are produced without tunneling process.

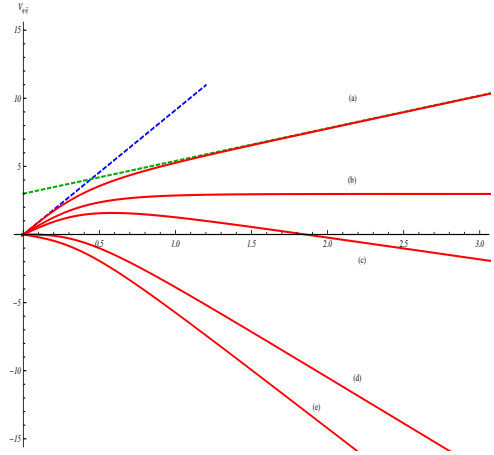


Figure 10. Quark-antiquark potentials $V_{q\bar{q}}$ are calculated as a function of the distance l between quarks with $r_0 = R = 1$ and $r_{\max} = w_0(0) = 3$ for $E = 0, 2.4, 4, 9.14, 11$ as (a), (b), (c), (d) and (e) respectively. Here $\tilde{\tau}_{\text{SD}} = E_c = 9.14$ and $\tilde{\tau}_{\text{QCD}} = 2.4$. The dashed blue line and dashed green line represent the tangential line $\tilde{\tau}_{\text{SD}}l$ at the origin and $\tilde{\tau}_{\text{QCD}}l + 1.5$ at the large l for $V_{q\bar{q}}$ with $E = 0$ respectively.

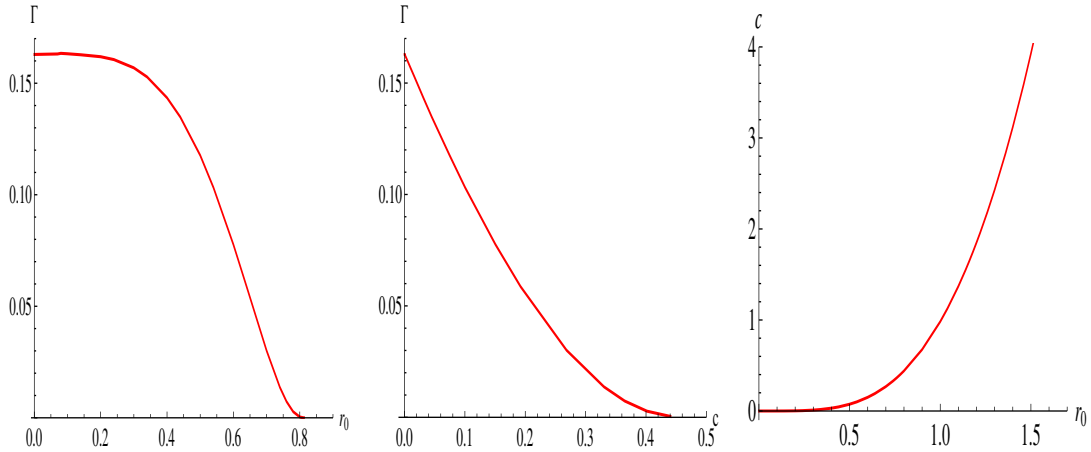


Figure 11. Γ for $E = 2.0$, $m_q = 1.0$ and $R = 1$ are shown as the function of r_0 (left) and of c (middle). The right figure shows the relation of c and r_0 .

5.1 Field condensate and Γ

As in the finite temperature case, the production rate Γ depends on the chiral condensate c , which is given by the above profile function $w_0(\rho)$ as

$$w_0(\rho) = m_q + \frac{c}{\rho^2} + \dots, \quad (5.18)$$

where m_q denotes the current quark mass.

The estimated value of Γ as a function of c is shown in the figure 11 for $E = 2.0$ and $m_q = 1.0$, where Γ is also shown as a function of the parameter r_0 . Γ decreases rapidly with c . This is reasonable since the effective mass m_q^{eff} increases with c as shown below,

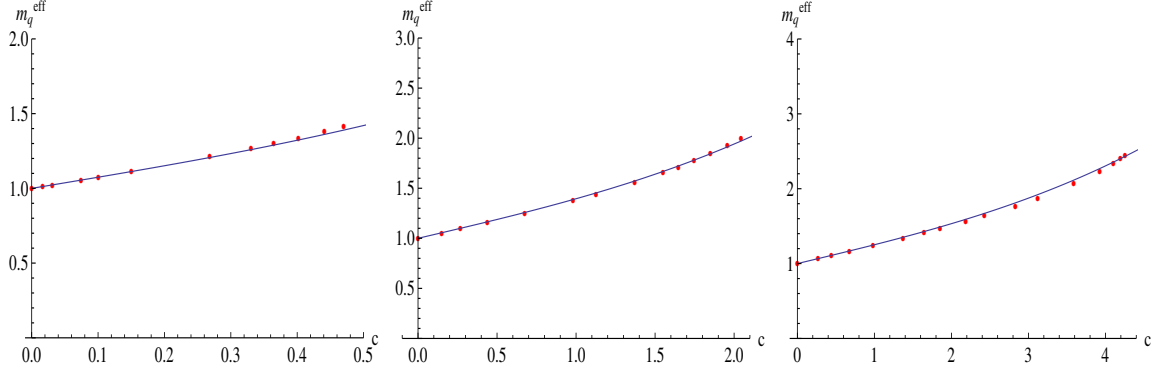


Figure 12. Dots represent the m_q^{eff} for $R = 1$ and $m_q = 1$ with $E = 2$, $E = 4$ and $E = 6$ from the right respectively. The solid line represents $m_q^{\text{eff}} = m_q + 2g_sc + 3h_2c^2 + 4h_3c^3$ with $g_s = 0.74/E$, $h_2 = 0$ and $h_3 = 1.61/E^4$.

and then Γ decreases. As for r_0 dependence, Γ is a decreasing function of r_0 . This point is understood since c increases with r_0 as shown in the figure 11. We notice that r_0 is related to the condensate of gauge field strength $\langle F_{\mu\nu}F^{\mu\nu} \rangle \propto r_0^4$, which constructs the string tension of the quark and anti-quark bound state.

5.2 m_q^{eff} and NJL coupling

As for the estimation of m_q^{eff} , we can perform it by changing the relation (4.19), which is given for the finite temperature case, as follows

$$\Gamma(m_q, c) \equiv \Gamma_c(m_q) = \Gamma_0(m_q^{\text{eff}}), \quad (5.19)$$

where the right hand side $\Gamma_0(m_q^{\text{eff}})$ is considered to be the same with the one used in (4.19) since the limit of $T = 0$ in the previous section and the limit of $r_0 = 0$ represent the same $\text{AdS}_5 \times S^5$ bulk metric.

By using the above relation (5.19), the m_q^{eff} is obtained as a function of c . The results are shown in the figure 12. Here m_q^{eff} runs from the current quark mass m_q to the upper bound $R\sqrt{E}$ as (4.7).

We compare our numerical results for m_q^{eff} with the effective quark mass from NJL model with higher order of c as follows,

$$m_q^{\text{eff}} = m_q + 2g_sc + 3h_2c^2 + 4h_3c^3. \quad (5.20)$$

By fitting the parameters numerically, we obtain the coefficients scaled by the appropriate powers of E as

$$g_s = 0.74/E, \quad h_2 = 0, \quad h_3 = 1.61/E^4. \quad (5.21)$$

Here m_q^{eff} can be fitted by including up to the term c^3 . However, in this case, h_2 is not obtained in the expected form $\text{const.}/E^{5/2}$, which is scaled by the canonical power of E stated in the section 3 through (3.10) and (3.11). Thus we get the reasonable solution such that $h_2 = 0$ as (5.21). This result seems to be inconsistent with the NJL model proposed

with higher order multi quark interactions in [26]. We should notice that c^2 term in [26] comes from six fermion interactions, which is introduced as the $U(1)_A$ breaking term with three flavor NJL model [27]. However, in our model, the flavor number N_f is set as one. Thus, our result is consistent with this fact.

6 Summary and discussion

In the D3/D7 brane system, a holographic Schwinger effect is studied by imposing an external electric field E for $\mathcal{N} = 2$ supersymmetric theory and also for two non-supersymmetric one, deconfining finite temperature and confining chiral symmetry breaking theory. In the present approach, the quark pair production rate Γ is given as the imaginary part of the on-shell D7-brane action.

For the deconfining and chiral symmetric theory at finite temperature, the dual bulk background is given by AdS_5 -Schwarzschild $\times S^5$ space-time. At zero temperature limit, the theory is reduced to the $\mathcal{N} = 2$ supersymmetric theory, and we obtain the analytic form of production rate, $\Gamma_0(m_q)$. The production rate obtained in this way is different from the one found via tunneling process as Schwinger effect in QED. Our result gives a production rate via a vacuum decay process.

From a dynamical viewpoint, this point can be understood more precisely. In the present holographic theory, the lower bound E_c corresponds to the tension of the linear potential, which is observed near very small distance between the quark and anti-quark for $E = 0$. This is found in terms of the string whose endpoints on the D7-brane are at the minimum point of r . This potential could make the bound state of the quark and anti-quark as mesons. The role of the imposed E is to reduce this attractive force. Actually, the condition, $E \geq E_c$, is sufficient to remove the attractive potential. So any tunneling phenomenon can not be expected in this case for getting the pair production rate of quarks. As a result, the quark and anti-quark will be separated without any resistance under the imposed E .

For the finite temperature case, there exist two types of D7-brane embeddings, the Minkowski and the BH types. For a fixed m_q , they are realized at the low and high temperature respectively. The transition of the embedding type occurs at the temperature T_c , where we can see a gap of Γ as shown in figure 6. As for the value of E_c , it is finite for the Minkowski type. On the other hand, it vanishes in the case of BH type. This is understood as the reflection of the screening of the attractive force due to the thermal fluctuation. Actually the meson states in the BH embeddings are observed as the quasi-normal modes [28]–[32] which have complex frequency. Therefore they are unstable. This is the reason of $E_c = 0$ in this case.

Thus it is easier to create the quark pair in the phase of black hole type than in the case of the Minkowski type. This point is assured by the temperature dependence of Γ , which increases rapidly near the transition temperature from the Minkowski to the BH embedding. Above the critical temperature, Γ slowly approaches to the $\Gamma_0(0)$ with increasing temperature. In other words, the explicit breaking of the chiral symmetry due to the mass term is also restored at $T = \infty$ from the viewpoint of the effective quark mass m_q^{eff} since m_q^{eff} for all m_q approaches to zero.

The characteristic feature of the theory is found in m_q^{eff} . Here m_q^{eff} is pulled out by identifying $\Gamma_T(m_q)$ with $\Gamma_0(m_q^{\text{eff}})$. The latter is the one of the supersymmetric theory dual to the $\text{AdS}_5 \times S^5$ bulk and its analytic form is given. Through this procedure, we could see how m_q^{eff} decreases with T . This behavior of m_q^{eff} is also related to the chiral condensate c , which is negative finite and decreasing with T . For the Minkowski embedding, heavy meson states are still living in spite of the fact that the theory is in the deconfinement phase. So, we have tried to understand the behavior at finite T from the NJL model by extending a simple mean field approximation. However the problem is not so simple. This problem is left as a challenging task to build an effective NJL type model at finite temperature deconfining theory.

In the next, we extended the analysis to a quark confining and chiral symmetry breaking phase. As in the supersymmetric case, the lower bound E_c is determined by a linear part of the potential at short distance, which is observed near very short distance between quark and anti-quark. The important point is that the other linear potential is seen also in the long distance region in the present case. This point is different from the case of the finite temperature deconfining phase. However, the E_c is not altered since the tension of the short range force is larger than the one of the long range confining force as shown by (5.17).

The confinement of the present theory is supported by the gauge field condensate $\langle F_{\mu\nu} F^{\mu\nu} \rangle \propto r_0^4$ which is characterized by the parameter r_0 . In this theory, the chiral symmetry is also broken. The order parameter c is therefore positive finite and increases with r_0 .

We find that the production rate Γ is decreasing with r_0 as expected since the effective quark mass m_q^{eff} increases with c . In this case, the effective mass m_q^{eff} is obtained by comparing Γ at finite r_0 and the one at $r_0 = 0$. We notice Γ for $r_0 = 0$ is reduced to the one given at zero temperature $\text{AdS}_5 \times S^5$ background. From the viewpoint of chiral symmetry breaking phase, we studied the relation between the effective quark mass m_q^{eff} and chiral condensate c . In this case, interestingly, we could get a consistent result with a simple NJL model with 4-th and 8-th terms of fermions.

Finally we comment on the tunneling process in the present D3/D7 case. We could find the tunneling via instanton configurations for $0 < E < E_c$ in the deconfining phase for a fixed m_q . In the confining phase, we would need a new lower bound $E_s (= \tilde{\tau}_{\text{QCD}})$ to overcome the confining force at long distance in order to realize the tunneling pair production. Then the tunneling process is found for $E_s < E < E_c$. In the case of $E > E_c$, on the other hand, we will find the tunneling production rate by using the real part of the D7 brane action. we will discuss these points in the future paper.

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